AP Calculus 2026-2027
Summer Assignment

Congratulations! You have been accepted into Advanced Placement Calculus AB for the next school year. This course will count as a math credit at SAINTS and you may also earn college credit if you pass the international AP Calculus AB exam in May 2027

Remember that by enrolling in this course, you are making a commitment to excellence in daily work. Successful students in AP Calculus possess the following characteristics:

- daily review of new content material taught in class
- diligent completion of homework on a daily basis
- participation in study groups or working with a study buddy
- understanding concepts vs. cramming details
- organizing notes and materials, grouping of similar concepts, discerning differences between concepts, and knowing and understanding major theorems and concepts
- asking questions in class and out of class before the next concept is introduced
- proficient in translating mathematical expressions into necessary and sufficient English justifications
- staying consistently diligent the entire year

It is a requirement to have a graphing calculator for this course and the exam. A TI-84 is recommended.

The problems in this packet are designed to help you review topics from previous mathematics courses that are important to your success in AP Calculus AB.

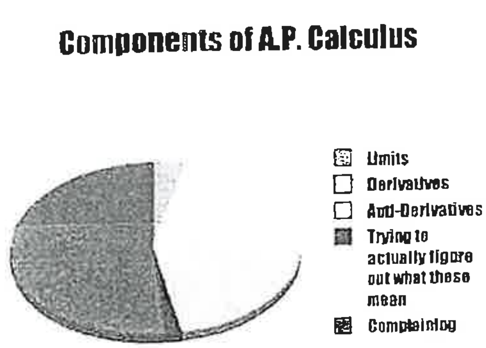
Guidelines:

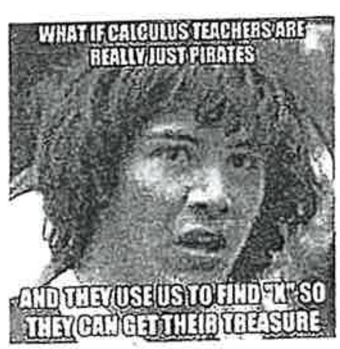
- Do <u>NOT</u> use a calculator on ANY of the problems. (TRY NOT TO)
- You may use previous notes or online tools to help you.
- Complete the packet with integrity.
- It is best to complete one to two weeks before school begins so the concepts are fresh in your mind.
- No class time will be used to work on this packet.

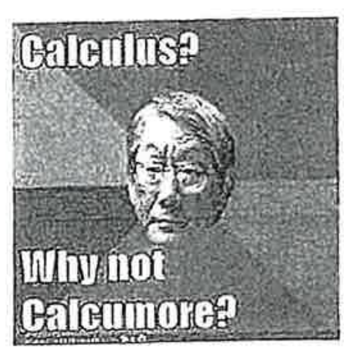
DUE: FIRST DAY OF CLASS

Please see the reverse side for helpful video links. If you still have questions, feel free to email me. PLEASE WATCH THE VIDEOS TO HELP YOU WITH THIS PREREQUISIT KNOWLEDGE!

I look forward to an exciting year of Calculus!







Helpful Videos

Below are links to videos that would be helpful when completing the summer assignment.

Please utilize them as the material in this packet WILL NOT BE REVIEWED OR TAUGHT IN CLASS.

Factoring:
http://tinyurl.com/Factoring-AllMethods

Solving Quadratic equations:
http://tinyurl.com/Quadratic-Equations-Solving

Logs and Exponentials:
http://tinyurl.com/Logs-Exponentials

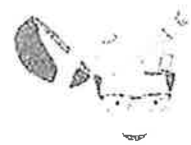
Rational and Radical Expressions and Equations
http://tinyurl.com/Radicals-Rational

Function Notation and Operations
http://tinyurl.com/Function-Notation-Operations

Evaluating Trig and Solving Trig Equations
http://tinyurl.com/Trig-Evaluating-Equations

Additional Helpful videos that include linear functions, absolute value, piecewise functions, asymptotes, etc.
http://tinyurl.com/CalcHelpfulVids

Limits and Continuity
http://tinyurl.com/Limits-Continuity-Vids



The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.

LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope = undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\qquad (xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\qquad \sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1 \ (x \neq 0)$ $\qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers.

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$\csc x = \frac{1}{\sin x}$ $\qquad \sec x = \frac{1}{\cos x}$ $\qquad \cot x = \frac{1}{\tan x}$ $\qquad \tan x = \frac{\sin x}{\cos x}$ $\qquad \cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$ $\qquad \tan^2 x + 1 = \sec^2 x$ $\qquad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$ $\qquad \cos(2x) = \cos^2 x - \sin^2 x$ **or** $1 - 2\sin^2 x$ **or** $2\cos^2 x - 1$

Math 150 – Pre-Calculus
Final Exam Review (Fall 2024)

Part I. Multiple Choice: Choose the best possible answer.

1. Find the domain of the function: $f(x) = \sqrt{2x+10} - 4$

   a. $(-\infty,-5] \cup [-4,\infty)$ b. $(-5,-\infty)$ c. $[-5,\infty)$ d. $[-4.,\infty)$

2. Find the distance between the points: $P = (-7,3)$ & $Q = (4,5)$

   a. $\sqrt{13}$ b. $5\sqrt{5}$ c. $\sqrt{17}$ d. $\sqrt{17}i$

3. Which of the following equations represent y *as a function* of x?

   a. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ b. $y = 3x^2 + 9$ c. $|y| = x - 10$ d. $x^2 + y^2 = 16$

4. Find the average rate of change of the function $f(x) = x^2 - 2x + 8$ from $x_1 = 2$ to $x_2 = 5$.

   a. 5 b. 3 c. -5 d. 11

5. Is $f(x) = x^4 - 2x^2 + 3$ even, odd, or neither? Does it have any symmetry?

   a. Odd with origin symmetry b. Even with x-axis symmetry

   c. Neither with no symmetry d. Even with y-axis symmetry

6. Find the inverse function $f^{-1}(x)$ of $f(x) = \frac{x+1}{x-2}$

   a. $f^{-1}(x) = \frac{2x-1}{x+1}$ b. $f^{-1}(x) = \frac{-2x+1}{x-1}$

   c. $f^{-1}(x) = \frac{x-2}{x+1}$ d. $f^{-1}(x) = \frac{2x+1}{x-1}$

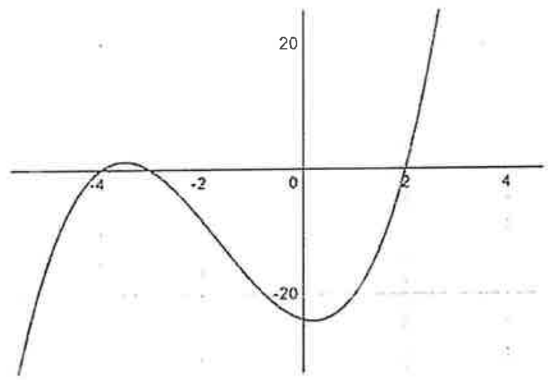
7. Solve for x: $2x^2 - 5x + 3 \geq 0$

   a. $(-\infty,1] \cup \left[\frac{3}{2},\infty\right)$ b. $\left[1,\frac{3}{2}\right]$ c. $(-\infty,1) \cup \left(\frac{3}{2},\infty\right)$ d. $\left(1,\frac{3}{2}\right)$

8. Given the following piecewise function, find $f(1)$: $f(x) = \begin{cases} x^3 - x & x \le 1 \\ x + 4 & x > 1 \end{cases}$

a. 3 b. 5 c. 2 d. 0

9. Find the equation of the polynomial graphed below.



a. $y = (x-2)^2(x+3)(x+4)$ b. $y = (x+2)^2(x-3)(x-4)$

c. $y = (x-2)(x+3)(x+4)$ d. $y = (x+2)(x-3)(x-4)$

10. Find the equation of the secant line for $f(x) = x^2 + 3x + 1$ on the interval $[2, 7]$.

a. $y = 2x - 13$ b. $y = 12x - 13$

c. $y = 12x - 130$ d. $y = 12x - 17$

Given $\boldsymbol{u} = \langle -2, 5\rangle$ and $\boldsymbol{v} = \langle -1, 8\rangle$, answer questions 11 and 12.

11. Find the dot product: $\boldsymbol{u} \cdot \boldsymbol{v}$

a. -38 b. 41 c. 42 d. 60

12. Find the angle between the vectors to the nearest tenth of a degree:

a. 138.4° b. 14.7° c. 36.2° d. 14.67°

13. Expand the expression by using the properties of logarithms:

$$log_2\left(\frac{4m\sqrt{n}}{p^2}\right)$$

a. $log_2 4 + log_2 m + \frac{1}{2} log_2 n - 2 log_2 p$

b. $log_2 4m + \frac{1}{2} log_2 n - 2 log_2 p$

c. $log_2 4 + log_2 m + \frac{1}{2} log_2 n + 2 log_2 p$

d. $2 + log_2 m + \frac{1}{2} log_2 n - 2 log_2 p$

14. Given $\cos u = -\frac{2}{7}$ and $\frac{\pi}{2} < u < \pi$, find $\cos\frac{u}{2}$ and $\sin 2u$.

a. $\cos\frac{u}{2} = -\frac{\sqrt{70}}{14}$

$\sin 2u = \frac{6\sqrt{5}}{7}$

b. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$

$\sin 2u = -\frac{6\sqrt{5}}{7}$

c. $\cos\frac{u}{2} = \frac{3\sqrt{14}}{14}$

$\sin 2u = -\frac{12\sqrt{5}}{49}$

d. $\cos\frac{u}{2} = \frac{\sqrt{70}}{14}$

$\sin 2u = -\frac{12\sqrt{5}}{49}$

For problems 16 and 17, let $\sin A = -\frac{7}{25}$ with A in Quadrant III and $\cos B = -\frac{4}{5}$ with B in Quadrant III.

15. Find $\sin(A + B)$

a. $-\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{4}{5}$ d. $-\frac{3}{5}$

16. Find $\tan(A - B)$

a. $\frac{100}{117}$ b. $-\frac{44}{75}$ c. $\frac{44}{75}$ d. $-\frac{44}{117}$

17. Simplify the trigonometric expression: $\frac{\sec\theta - 1}{1 - \cos\theta}$

a. $\sec\theta$ b. $\cos\theta$ c. $\frac{\sec\theta + \cos\theta}{\sin^2\theta}$ d. -1

18. Simplify the trigonometric expression: $\frac{1}{\cos x+1}+\frac{1}{\cos x-1}$

a. $\sec x$ b. $-2\csc x\cot x$ c. $-2\csc^2 x$ d. $\frac{2}{\cos^2 x-1}$

19. Evaluate $\tan^{-1}(-1)$

a. $\frac{3\pi}{4}, \frac{7\pi}{4}$ b. $\frac{7\pi}{4}$ c. $-\frac{\pi}{4}$ d. $\frac{3\pi}{4}$

20. Evaluate $sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right)$

a. $\frac{\sqrt{3}}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{3}\pi}{2}$

21. In triangle ABC, if $a = 3.7\ cm, c = 6.4\ cm, and\ B = 23°$, find b.

a. 4.1 cm   b. 3.3 cm   c. 5.7 cm   d. 11.1 cm

Part II. Short Answer Section: Show your work.

1. Find the difference quotient: $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ for $f(x) = 5x - x^2$.

2. The graph of $f(x) = \sqrt{x}$ is shifted up 3 units, right 4 units, reflected about the y-axis, and compressed vertically by a factor of 3. Name the resulting function g, then write an equation for $g(x)$ in terms of f.

3. Describe the transformations that would produce the graph of g from the graph of f. Be specific and detailed. If more than one transformation is needed, specify the order in which the transformations should be applied.

a. $g(x) = f(2x) + 7$

b. $g(x) = -f(x-3) - 5$

4. Find the compositions $(f \circ g)(x)$ and $(g \circ f)(x)$ using the following functions: $f(x) = \sqrt[3]{x-5}$ and $g(x) = x^3 + 1$. Are $f(x)$ and $g(x)$ inverse functions of each other? Explain why or why not.

5. Given the function. $f(x) = x^3 + 2x^2 + 4x + 8$.

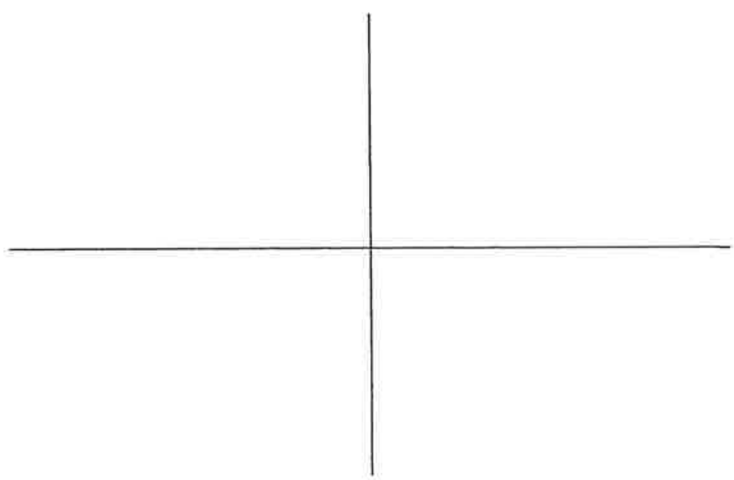
a. Factor the polynomial over the real numbers as the product of linear factors or irreducible quadratic factors.

b. State all zeros (real and imaginary) and their associated multiplicities.

c. State the end behavior.

d. Find the y-intercept.

e. Using the information in parts a – d above as a guide, sketch $f(x)$.



6. Find all asymptotes that exist (vertical, horizontal and/or slant) for the following function: $f(x) = \frac{-4x^2+1}{x^2+x-2}$. If they do not exist, explain why. State the domain for the function.

Vertical | Horizontal | Slant

Domain:

7. Solve the following exponential and logarithmic equations. Leave answers in exact form:

a. $8^x = 32^{x-1}$

b. $5^x + 8 = 26$

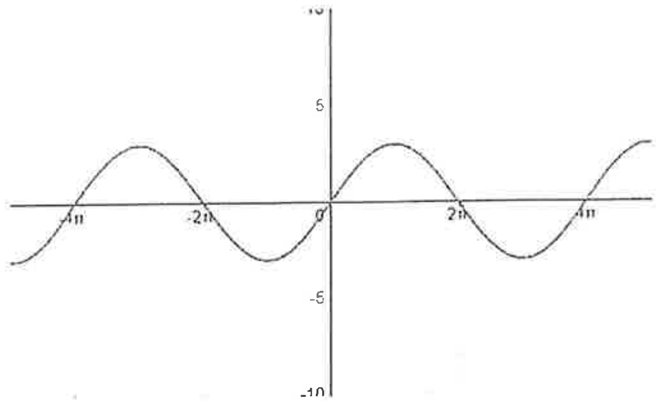
c. $log_2 x + log_2(x+2) = log_2(x+6)$

d. $log(8x) - log(x+1) = 2$

8. Given $\sin\theta = -\frac{12}{15}$ and θ terminates in Quadrant III, find the five remaining trigonometric functions of θ.

9. Solve the following non-linear system: $\begin{cases} x - 2y = -6 \\ x^2 - y = 0 \end{cases}$ by substitution.

10. Write a trigonometric function matching the graph below using a **sine** function:



11. Given the following trigonometric function: $y = 2 + 2\sec(x - \frac{\pi}{4})$

Find the period, amplitude, horizontal translation, and vertical translation.

Amplitude:

Period:

Horizontal Translation:

Vertical Translation:

12. Solve the trigonometric equations:

a. $\csc^2 x + 3\csc x - 4 = 0$ over $[0, 2\pi)$

b. $2\sin^2 x + 5\cos x - 4 = 0$

c. $2\sin 2x + \sqrt{3} = 0$ over $[0, 2\pi)$

d. $\sec 4x - 2 = 0$

2. Determine all values of x that satisfy each equation below.

(a) [5 pts] $\dfrac{1}{x+3} = \dfrac{5}{2x-1}$

(b) [5 pts] $\sqrt{x^2 - x} = 2x + 1$

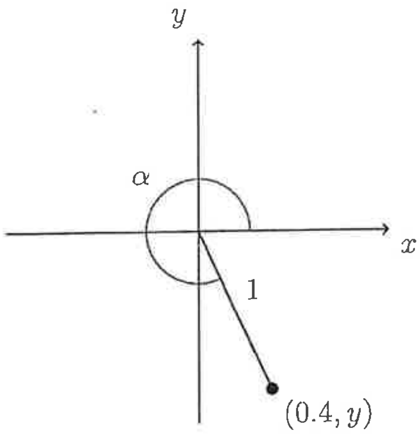
(c) [5 pts] $\dfrac{e^{3x}+1}{5}=7$

(d) [5 pts] $\log_2(7-3x)=2+\log_2(x+7)$

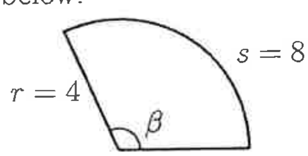
(e) [5 pts] $6 \cdot 5^x = 8 \cdot 12^{9-4x}$

3. Determine the value of each of the requested quantities below. If an exact number is not requested, numerical values should be to within 0.01 of the true value. (**All angles are given in radians unless otherwise stated and your answer should be expressed in radians if you have to determine their numerical value.**)

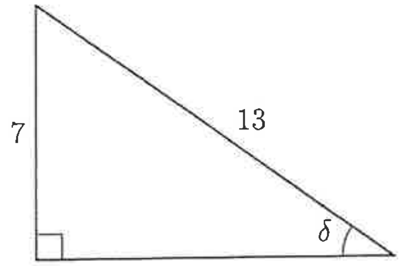
(a) [5 pts] Determine the cosine, sine, and tangent of the angle α in the diagram below. The coordinate, $(0.4, y)$, is a point on the unit circle.



(b) [5 pts] Determine the radian measure of the angle β in the sector shown in the diagram below:



(c) [5 pts] Determine the sine, cosine, and tangent of the angle δ in the diagram below.



(d) [5 pts] Determine the **exact** numerical value of the following expression and express it without the use of any trigonometric functions:

$$\cos\left(\arctan(1.7) + \arcsin(0.01)\right).$$

(Show your work and do not provide a numerical estimate or use a calculator.)

4. Answer each of the following questions using the functions defined below:

$g(x) = \sqrt{x}$, $h(x) = \ln(2x - 1)$

(a) [5 pts] Determine the inverse of $h(x)$.

(b) [5 pts] Determine the value of $g\left(h\left(1\right)\right)$. (Show your work and each step. Do not simply write down the final number.)

(c) [5 pts] Determine the domain of $h(x)$. (Your answer should be an interval. Use interval notation.)

(d) [5 pts] Determine the domain of $h(g(x))$. (Your answer should be an interval. Use interval notation.)

5. Two functions are defined to be

$$\begin{aligned} f(x) &= 3e^{2x}, \\ g(x) &= e^{ax}, \end{aligned}$$

where a is a constant.

(a) [5 pts] What values of a will guarantee that the function $g(x)$ will be an increasing function. (Your answer should be written as an interval.)

(b) [5 pts] What values of a will guarantee that the graphs intersect at $x = 2$?

6. The function $l(x)$ is a linear function whose graph includes the points $(3, 4)$ and $(-1, 8)$. The function $q(x)$ is a quadratic function whose maximum on its graph is located at the point $(2, 5)$, and the y-intercept of the graph is located at $(0, -7)$.

   (a) [5 pts] Determine the formula for $l(x)$.

   (b) [5 pts] Determine the formula for $q(x)$.

   (c) [5 pts] Determine a line parallel to $l(x)$ that includes the coordinate $(1, -1)$.

   (d) [3 pts] Suppose that the function $q(x)$ had a different y-intercept. What y-intercept would result in a function that is not a quadratic?

7. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

   (a) [5 pts] The total cost of risk for a project increases by 1.4% each year. *The total cost of risk as a function of time over multiple years.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

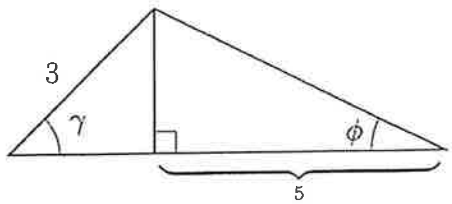
   (b) [5 pts] The total cost of risk for a project increases during warmer months and decreases during colder months. *The total cost of risk as a function of time over multiple years.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

   (c) [5 pts] The total cost of risk for a project increases by $8,000 each year. *The total cost of risk as a function of time over multiple years.*

Linear Function	Quadratic Function	Exponential Function	Trigonometric Function

8. [10 pts] Determine the radian measure of the angle ϕ in the diagram below. The radian measure of the angle γ is $\frac{\pi}{4}$. (If you use a calculator to provide an approximation, the approximation should be within two decimal places.)



9. Determine the formulas for the functions described in each statement below.

   (a) [5 pts] An institution offers a savings account that has an 1.3% annual interest compounded monthly. An account is opened with an initial balance of $3,000. Determine the function that gives balance of the account given the number of years after it is opened.

   (b) [5 pts] An exponential function, $p(x) = Ce^{rx}$, whose graph includes the points $(0, 4)$ and $(5, 8)$.

   (c) [5 pts] The function whose inverse is $g^{-1}(x) = \dfrac{1}{1+x}$.

10. A sled slides down a hill and its speed increases. When it reaches the bottom, the path levels out, and the speed decreases. The speed of the sled is approximated by the following function of the time (seconds),

$$v(t) = \begin{cases} 1.7t & 0 \leq t \leq 5, \\ 8.5e^{-0.3(t-5)} & t > 5. \end{cases}$$

(a) [5 pts] Is the function a one-to-one function? (Briefly explain your reasoning. You can use algebraic, graphical, or an informal written argument to justify your conclusion.)

(b) [5 pts] What is the maximum velocity?

(c) [5 pts] Determine the average rate of change in the speed from the initial time ($t = 0$ seconds) and $t = 10$ seconds. (If you use a calculator to provide an approximation, the approximation should be within two decimal places.)

11. When a drug is administered to a patient there is a probability that the patient will experience an adverse reaction. It is assumed that the probability depends on the dosage (in milligrams) given to the patient. One model that is used to approximate the probability, p, given the dosage, x mg, is

$$m \cdot x + b \;=\; \ln\left(\frac{p}{1-p}\right),$$

where m and b are constants[1].

(a) [5 pts] In experiments it is estimated that if the dose is 0.03mg then the probability of an adverse reaction is approximately 0.3, and if the dose is 0.05mg then the probability of an adverse reaction is approximately 0.6. Determine the values of m and b.

(b) [5 pts] Use your results to determine the function that represents the probability of an adverse reaction given the dosage. If you are unsure of your results in the previous part assume $m = 50$ and $b = -2$. (Explicitly state you are using these values.)

[1] Michael E. Ginevan, Deborah K. Watkins, Logarithmic dose transformation in epidemiologic dose–response analysis: Use with caution, Regulatory Toxicology and Pharmacology, Volume 58, Issue 2, 2010, Pages 336-340, https://doi.org/10.1016/j.yrtph.2010.07.007

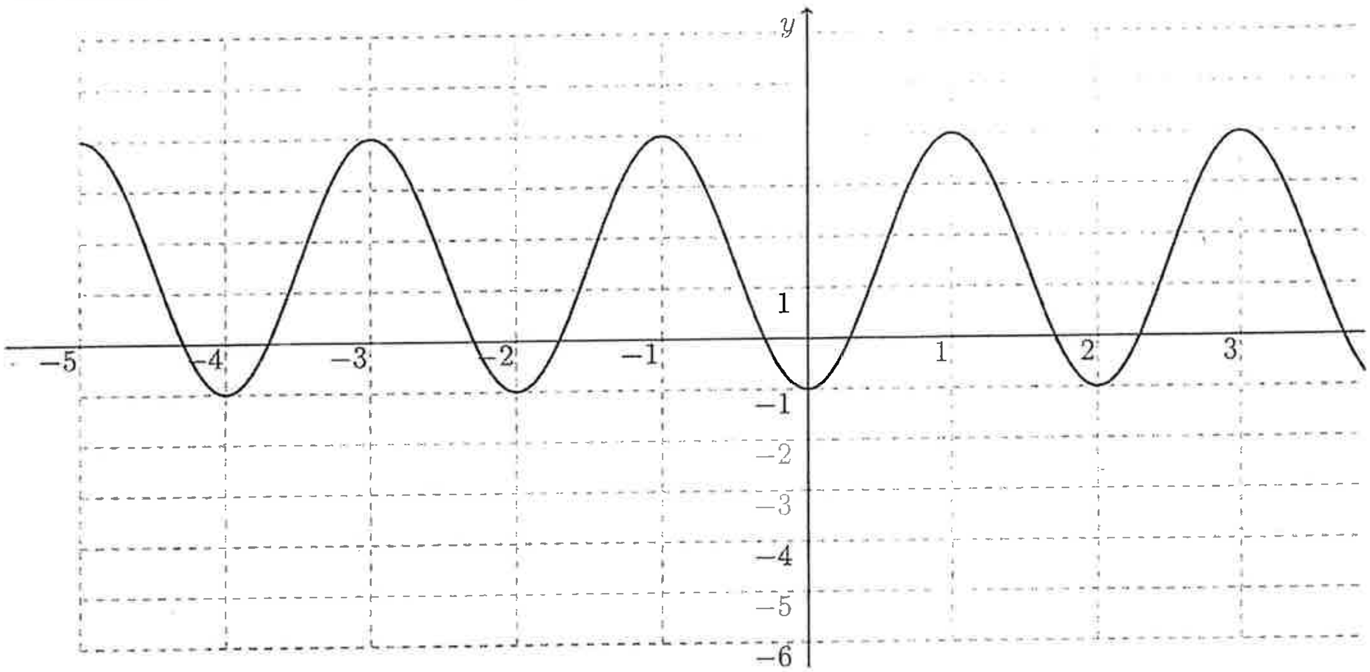
12. [10 pts] Verify the following identity,

$$(\cos(\alpha) - \cos(\beta))^2 + (\sin(\alpha) - \sin(\beta))^2 = 2 - 2\cos(\alpha - \beta)$$

13. [10 pts] Determine a formula for the function whose graph is shown below expressed as a cosine function,

$$k(x) \quad = \quad A\cos(bx + c) + d.$$

The values of A and b should be positive numbers.



$A =$

$b =$

$c =$

$d =$

14. [10 pts] There are 12 hours of daylight available to a bird. Suppose, the bird only takes part in two activities, foraging and defending its territory. The bird will fill the whole 12 hours with only those two activities, and each activity has an energy cost associated with it:

Foraging The energy cost of foraging is 7 times the amount of time (in hours) spent foraging (i.e. 7*time foraging). For example, if the bird spends 3 hours foraging the energy cost is 21 energy units.

Defending Territory To defend its territory the bird must fly around a large area, and the energy cost of defending its territory is the square of the time spent defending its territory (i.e. $(\text{time defending})^2$). For example, if the bird spends three hours defending its territory, the cost is 9 units.

Determine the amount of time the bird should spend foraging and the time defending its territory that will **minimize** the bird's total energy cost.

2. Determine all values of x that satisfy each equation below.

(a) [5 pts] $\sqrt{4x+1} = \frac{1}{2}$.

(b) [5 pts] $\frac{3}{2-x} = 5x$.

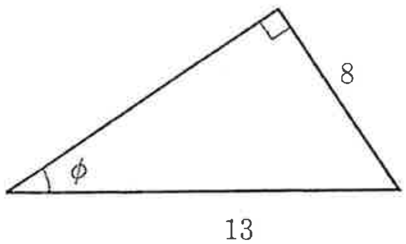
(c) [5 pts] $\log_3(3+\sqrt{x})=4$.

(d) [5 pts] $(8-e^x)^5=32$.

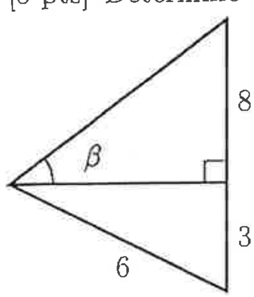
(e) [5 pts] $4 \cdot 13^{2-x} = 14 \cdot 7^{x-1}$.

3. Determine the value of each of the requested quantities below. If an exact number is not requested, numerical values should be to within 0.01 of the true value. (**All angles are given in radians unless otherwise stated and your answer should be expressed in radians if you have to determine its numerical value.**)

   (a) [5 pts] Determine the sine, cosine, and tangent of the angle ϕ as shown in the diagram below:



   (b) [5 pts] Determine the radian measure of the angle β in the diagram below.



(c) [5 pts] Verify the equality

$$(\cos(\theta) + \sin(\theta))^2 + (\cos(-\theta) + \sin(-\theta))^2 \;=\; 2.$$

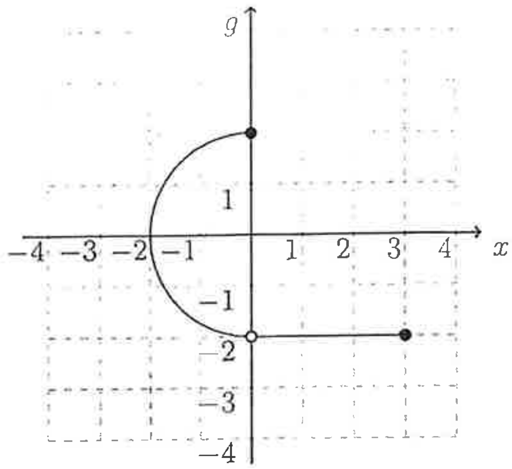
(Show every step, and do not make more than one algebraic step at a time.)

(d) [5 pts] Determine the **exact** numerical value of the following expression and express it without the use of any trigonometric functions:

$$\sin\left(\arctan(0.1) + \arccos(-0.4)\right).$$

(Show your work and do not provide a numerical estimate or use a calculator.)

4. Three relationships, g, h, and k, are defined below. Use these definitions to answer each of the questions that follow.



x	h
-1	2
0	5
1	3
2	3

$$k(x) = x^2 + 1.$$

(a) [5 pts] For each relationship (g, h, and k) determine whether or not the relationship is a function. Briefly justify your conclusion. (If you use a "named" test, state the name of the test and explicitly discuss how it implies the relationship is a function.)

g

h

k

(b) [5 pts] For each relationship that is a function, determine if the function is one-to-one. (Briefly justify your conclusion and do not simply state the name of a test.)

g

h

k

5. Two functions are defined by

$$\begin{aligned} m(x) &= 4^x + 1, \\ p(x) &= (1+b)^x. \end{aligned}$$

(a) [5 pts] Determine the domain and range of $m(x)$. State any asymptotes for the function $m(x)$.

(b) [3 pts] Determine the values of b that cause $p(x)$ to be an exponentially decaying function. (Your answer should be an interval. Use interval notation.)

6. Answer each question below using the function

$$r(x) = \frac{6}{2x+1}.$$

(a) [5 pts] Determine the inverse of the function.

(b) [5 pts] Determine the average rate of change of the function, $r(x)$, from $x = 0$ to $x = 5$.

7. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

   (a) [5 pts] The total mass of phytoplankton in an area of the ocean increases at a constant rate each day. *The total mass of the phytoplankton as a function of time over a month.*

   Linear Function — Quadratic Function — Exponential Function — Trigonometric Function

   (b) [5 pts] The total mass of phytoplankton in an area of the ocean increases by 1.3% each week. *The total mass of the phytoplankton as a function of time over several months.*

   Linear Function — Quadratic Function — Exponential Function — Trigonometric Function

   (c) [5 pts] The total mass of phytoplankton in an area of the ocean increases during the summer months and decreases during the winter months and repeats this cycle each year. *The total mass of the phytoplankton as a function of time over many years.*

   Linear Function — Quadratic Function — Exponential Function — Trigonometric Function

8. [10 pts] Doctor Bunsen Honeydew has a test tube with 0.7mg of a sensitive material in it, and he begins to gently sprinkle more of the chemical into the test tube at a constant rate of 0.15mg per minute. The test tube can hold 3.0mg of the material. Determine a function that returns the amount of material in the test tube given the time in minutes, and determine when the test tube starts to overflow. (Your function for the amount of material should only be for times before an overflow occurs.)

9. Each of the following questions refer to the function

$$d(x) = \begin{cases} -(x-1)^2+1 & 0 \leq x \leq 1.5, \\ -(x-2)^2+1 & 1.5 < x \leq 3.0, \end{cases}$$

(a) [5 pts] Determine the coordinates for all local maximum values of $d(x)$. (Briefly explain why your coordinates are a local maximum.)

(b) [5 pts] Determine the intervals where the function, $d(x)$, is increasing. (Your answer should be in interval notation.)

(c) [5 pts] Determine the domain and range of the function, $d(x)$.

(d) [2 pts] Is it possible to draw a sketch of the graph without lifting the pencil from the page?

10. A function is defined to be

$$s(x) = -3e^{rx} + C$$

where $s(0) = 5$, and $s(6) = 7.5$.

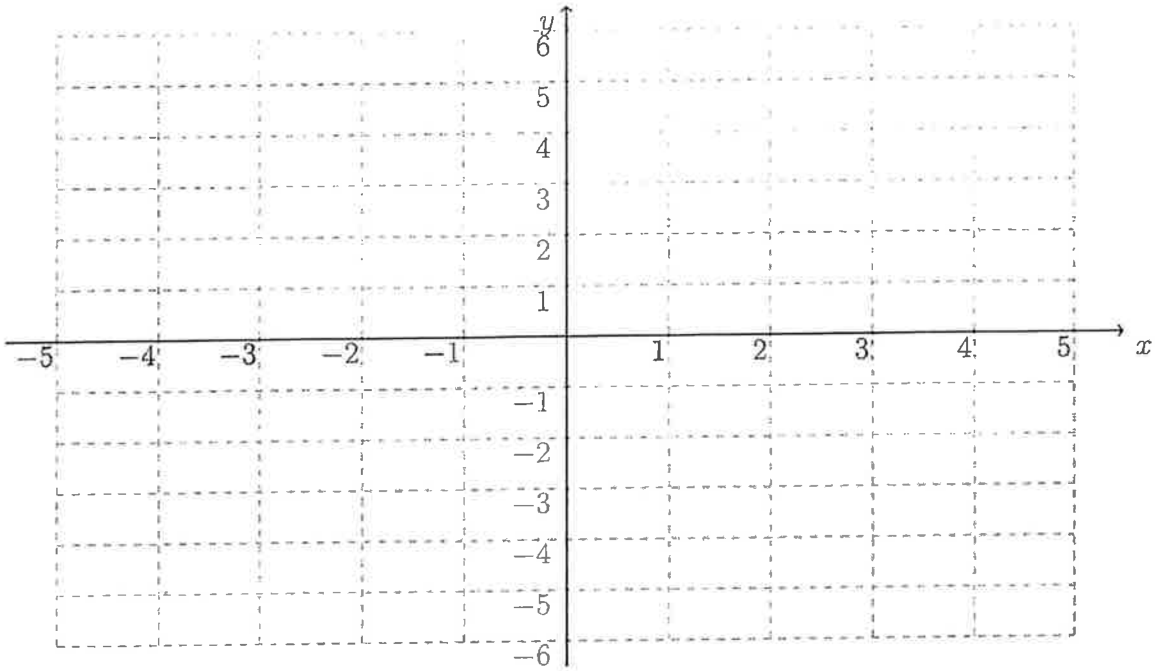
(a) [5 pts] Determine the values of the constants C and r.

(b) [5 pts] Is the function always increasing, always decreasing, or neither? (Briefly justify your answer.)

(c) [3 pts] Determine if the value of the function approaches a particular number as the value of x gets extremely large, and if so, determine the value.

11. [10 pts] Use the axes below to make a sketch of the graph of the function

$$w(x) = 3\sin\left(\frac{\pi}{3}x + \frac{4\pi}{3}\right) - 2.$$



12. A kudzu vine grows 3cm a day, and it climbs straight up the side of a building. A video camera will be placed on the ground directly in front of the vine. The camera will be 150cm away from the wall. The vine starts at 10cm above the ground when the camera is put in place. The angle of elevation for the camera will be constantly adjusted so that the top of the vine is in the center of the picture.

   (a) [5 pts] When the angle of elevation reaches $\frac{4\pi}{9}$ the height of the vine will be measured, and whoever has the closest guess will win a prize. What will the height be when this occurs?

   (b) [5 pts] Determine the angle of elevation as a function of time in days.

13. It is estimated[1] that the distance a locust can hop is related to the length of the animal's femur,

$$d = C \cdot l^{2/3},$$

where d is the distance of the hop in meters(m), C is a constant, and l is the length of the animal's femur in millimeters(mm). The length of a locust's femur (mm) is related to the animal's total mass,

$$l = 6 \cdot m^{1/3},$$

where m is the animal's mass in grams.

(a) [5 pts] The length of a given locust's femur is 7.5mm, and the distance it can hop is measured to be 0.05m. Determine the value of C.

(b) [5 pts] The distance that a different locust can hop is 0.06m. Use the relationships above and your estimate of C to determine the length of its femur. (If you do not feel confident in your calculation of C above, use a value of $C = 0.025$.)

(c) [5 pts] Determine the function that returns the mass of a locust given the distance it can hop. (Use the same value for C that you used above.)

[1] Julie M. Gabriel; The Development of the Locust Jumping Mechanism : I. Allometric Growth and its Effect on Jumping Performance. J Exp Biol 1 September 1985; 118 (1): 313-326.

14. [15 pts] An insect will move 100mm. The insect can either walk or it can run. The energy expenditure depends on whether it walks or runs.

Walking The energy expenditure when walking is the square of the distance walking. (i.e. (distance walking)^2). For example, if the insect walks 3mm, the expenditure is 9 units.

Running The energy expenditure when running is six times the square of the distance running. (i.e. 6(distance running)^2). For example, if the insect runs 3mm, the expenditure is 54 units.

Determine the distance the insect should walk and the distance it should run to **minimize** the insect's total energy cost.