AP Calculus AB 2024-2025

Summer Assignment

Congratulations! You have been accepted into Advanced Placement Calculus AB for the next school year. This course will count as a math credit at SAINTS and you may also earn college credit if you pass the international AP Calculus AB exam in May 2025.

Remember that by enrolling in this course, you are making a commitment to excellence in daily work. Successful students in AP Calculus possess the following characteristics:

- daily review of new content material taught in class
- diligent completion of homework on a daily basis
- participation in study groups or working with a study buddy
- understanding concepts vs. cramming details
- organizing notes and materials, grouping of similar concepts, discerning differences between concepts, and knowing and understanding major theorems and concepts
- asking questions in class and out of class before the next concept is introduced
- proficient in translating mathematical expressions into necessary and sufficient English justifications
- staying consistently diligent the entire year

It is a requirement to have a graphing calculator for this course and the exam. A TI-84 is recommended.

The problems in this packet are designed to help you review topics from previous mathematics courses that are important to your success in AP Calculus AB.

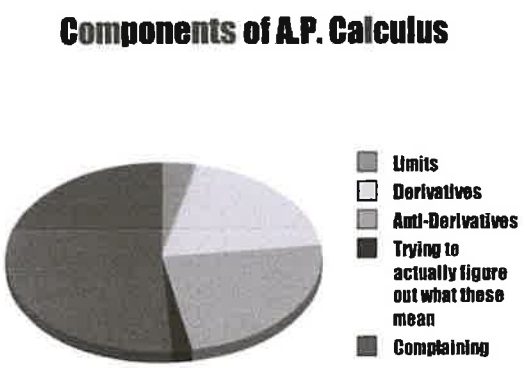
Guidelines:

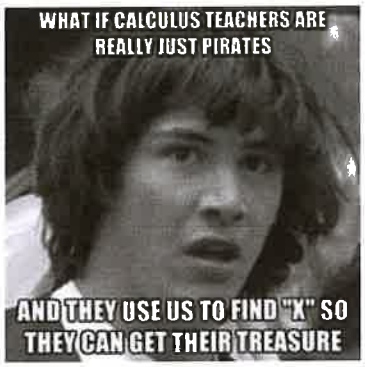
- Do NOT use a calculator on ANY of the problems.
- You may use previous notes or online tools to help you.
- Complete the packet with integrity.
- It is best to complete one to two weeks before school begins so the concepts are fresh in your mind.
- No class time will be used to work on this packet.

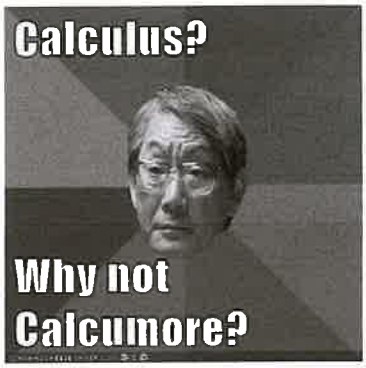
DUE: Wednesday August 14th, 2024

Please see the reverse side for helpful video links. If you still have questions, feel free to email me. PLEASE WATCH THE VIDEOS TO HELP YOU WITH THIS PREREQUISIT KNOWLEDGE!

I look forward to an exciting year of Calculus!







Helpful Videos

Below are links to videos that would be helpful when completing the summer assignment.

Please utilize them as the material in this packet WILL NOT BE REVIEWED OR TAUGHT IN CLASS.

Factoring:
http://tinyurl.com/Factoring-AllMethods

Solving Quadratic equations:
http://tinyurl.com/Quadratic-Equations-Solving

Logs and Exponentials:
http://tinyurl.com/Logs-Exponentials

Rational and Radical Expressions and Equations
http://tinyurl.com/Radicals-Rational

Function Notation and Operations
http://tinyurl.com/Function-Notation-Operations

Evaluating Trig and Solving Trig Equations
http://tinyurl.com/Trig-Evaluating-Equations

Additional Helpful videos that include linear functions, absolute value, piecewise functions, asymptotes, etc.
http://tinyurl.com/CalcHelpfulVids

Limits and Continuity
http://tinyurl.com/Limits-Continuity-Vids

The following formulas and identities will help you complete this packet. You are expected to know ALL of these for the course.

LINES

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Horizontal line: $y = b$ (slope $= 0$)

Vertical line: $x = a$ (slope = undefined)

Parallel → same slope

Perpendicular → opposite reciprocal slopes

QUADRATICS

Standard: $y = ax^2 + bx + c$

Vertex: $y = a(x - h)^2 + k$

Intercept: $y = a(x - p)(x - q)$

Parabola opens: up if $a > 0$
down if $a < 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXPONENTIAL PROPERTIES

$x^a \cdot x^b = x^{a+b}$ $\quad (xy)^a = x^a y^a$

$\frac{x^a}{x^b} = x^{a-b}$ $\quad \sqrt[n]{x^m} = x^{m/n}$

$x^0 = 1\ (x \neq 0)$ $\quad \left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$

$x^{-n} = \frac{1}{x^n}$

In general, it is fine to have negative exponents in your answers!

LOGARITHMS

$y = \log_a x$ is equivalent to $a^y = x$

$\log_b(mn) = \log_b m + \log_b n$

$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \log_b m$

TRIGONOMETRIC IDENTITIES

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$ $\quad \tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$ $\quad \tan^2 x + 1 = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$ $\quad \cos(2x) = \cos^2 x - \sin^2 x$ **or** $1 - 2\sin^2 x$ **or** $2\cos^2 x - 1$

Whenever you see a video icon, click it to watch a short video about the content. For example, this video link will help you with the next page.

AP Calculus AB Summer 2024-2025 Assignment

It's best to do this packet closer to the start of school! **DUE: Wednesday August 14th, 2024**

Do **NOT** use a calculator!

Name: _____ PD: _____ **SHOW ALL WORK ON THIS PACKET!**

Simplify #1-20 completely.

1. $\frac{3}{7}+\frac{5}{4}=$

2. $\frac{2}{9}-6=$

3. $\frac{\frac{8}{5}}{\frac{2}{3}}=$

4. $\frac{4}{\frac{3}{8}}=$

5. $\frac{3}{x-2}-\frac{1}{x+3}=$

6. $\frac{x+5}{x^2-2x-35}=$

7. $\frac{x^3-64}{x-4}=$

8. $\frac{7-x}{x^2-49}=$

9. $\frac{\sin^2 x+\cos^2 x}{\sec x}=$

10. $\cot x \sec x=$

11. $(x+y)^2=$

12. $\sqrt{75}=$

13. $\frac{4xy^{-2}}{6x^4y^{-3}}=$

14. $\sqrt{x+1}+\frac{2}{\sqrt{x+1}}=$

15. $5a^{\frac{4}{3}}2a^{\frac{1}{3}}=$

16. $\left(4a^{\frac{7}{3}}\right)^{\frac{3}{2}} =$

17. $10\sqrt{20} + 11\sqrt{45} =$

18. $\sqrt{112a^5} =$

19. $36^{\frac{1}{4}} \cdot 36^{\frac{1}{4}} =$

20. $\dfrac{\sqrt[6]{x^2}}{\sqrt[3]{x^5}} =$

Consider the following to answer #21-30.

x	2	4	7
$f(x)$	3	5	2

$h(x) = \{(2,1), (3,2), (7,4)\}$ $g(x) = 3x^2 + 1$ $k(x) = \sqrt{x+2}$

21. $(f + h)(2) = f(2) + h(2)$

22. $(k - g)(2) =$

23. $(f \circ h)(7) =$

24. $g(k(7)) =$

25. $k(g(3x + 2)) =$

26. $g^{-1}(x) =$

27. $\dfrac{f(4)}{g(2)} =$

28. $(g \cdot k)(x) =$

29. $g\big(k(x - 3)\big) =$

30. $f^{-1}(x) =$

x			
$f^{-1}(x)$			

For # 31-36, write the equation for the line given the information below.
Leave your answer in point-slope form (when possible). $y - y_1 = m(x - x_1)$

31. slope $= -\frac{2}{3}$ containing the point (2, 3).

32. containing the points (1, –3) and (–5, 2).

33. slope = 0 and containing the point (4, 3).

34. perpendicular to the line in #38 and containing the point (3, 4).

35. passing through the point (−4,2) with an undefined slope.

36. passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

For #44-46, evaluate the expression $\frac{f(x+h)-f(x)}{h}$ given *f(x)* below. Simplify as much as possible.

37. $f(x) = 9x + 3$ EXAMPLE OF SET UP: $\frac{(9(x+h)+3)-(9x+3)}{h}$

38. $f(x) = 5 - 2x$

39. $f(x) = x^2 + 2x$

For #40-44, determine all the points of intersection by solving the system of equations using substitution! *Write your answer as a coordinate point.* **Must have algebra (not guess and check) to receive credit.**

40. $5x - 2y = 5$
    $3x + y = 3$

41. $y^2 = 4x$
    $x + y = 3$

42. $y = x^3$
$y = x$

43. $xy = 4$
$x^2 + y^2 = 8$

44. $y = \sqrt{x-2}$
$y = x + 1$

For #45-52, solve for x. Show all work that leads to your solution.

45. $2x^2 + 9x + 7 = 3$

46. $\sqrt{x-10} - 4 = 0$

47. $(x+2)^2 = 12$

48. $3ax + 6ab = 7ax + 3ab$

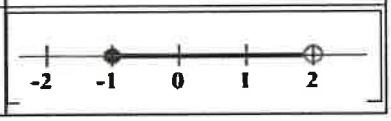
49. $6x^2 = 2x$

50. $|x - 5| = 3$

51. $x^2 - 5x = -2$

52. $\frac{a+3x}{b} = \frac{c}{2}$

For #53-55, complete the table using the appropriate notation or graphical representation.

	Solution	Interval Notation	Graph
53.	$-2 < x \leq 4$		
54.		$[-1, 7)$	
55.			

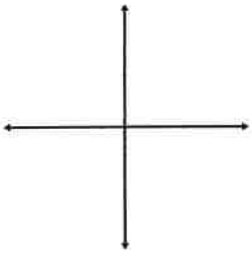
For #56-61, describe the transformation from the parent function and state the domain and range.

Function	Transformation	Domain	Range
56. $y = (x+1)^2$			
57. $y = \lvert x \rvert - 2$			
58. $y = \sqrt{x+3} - 4$			
59. $y = 2\ln x$			
60. $y = e^x$+5			
61. $y = x + 7$			

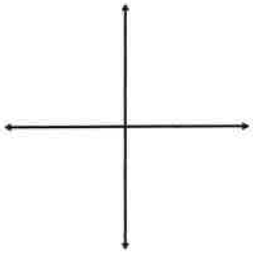
For #62-69, sketch the function. Label important features (x & y intercepts, vertex, asymptotes, etc).

NO Calculator!!!!!

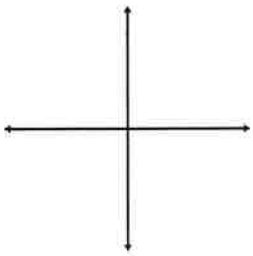
62. $y = \sin x \ [-2\pi, 2\pi]$



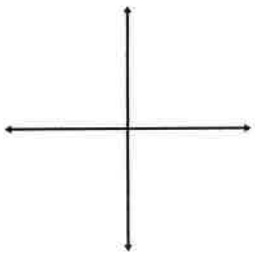
63. $y = x^2 + 2$



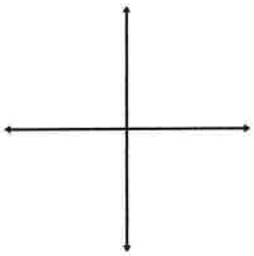
64. $y = \sqrt{x+1}$



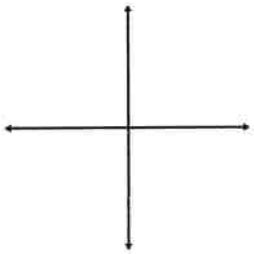
65. $y = x^3$



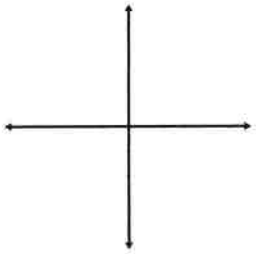
66. $y = \frac{1}{x}$



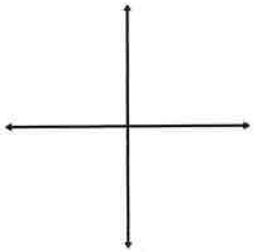
67. $y = \lvert x - 1 \rvert - 2$



68. $y = e^x + 4$



69. $y = \ln x$



For #70-72, determine the vertical asymptote(s) for the function. The vertical asymptote occurs where the function is undefined (where the denominator is equal to zero). *Your answer should be in the form x=#.*

70. $f(x) = \frac{1}{x^2}$

71. $f(x) = \frac{x^2}{x^2-4}$

72. $f(x) = \frac{2+x}{x^2(1-x)}$

For #73-75, determine the horizontal asymptote using the three cases below.
Your answer should be in the form y=#.

Case 1: The degree of the numerator is less than the degree of the denominator. The function has a horizontal asymptote at $y = 0$.

Case 2: The degree of the numerator is equal to the degree of the denominator. The function has a horizontal asymptote equal to the ratio of the leading coefficients.

Case 3: The degree of the numerator is greater than the degree of the denominator. The function has no horizontal asymptote.

73. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

75. $f(x) = \frac{4x^5}{x^2-7}$

74. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$

For #76-89, *without a calculator*, determine the exact value of each expression.

76. $\sin 0$ = _____

77. $\cos\pi$ = _____

78. $\cos\frac{3\pi}{4}$ = _____

79. $\csc\frac{3\pi}{4}$ = _____

80. $\cos\frac{5\pi}{3}$ = _____

81. $\tan\frac{7\pi}{4}$ = _____

82. $\tan\frac{3\pi}{2}$ = _____

83. $\tan\frac{\pi}{6}$ = _____

84. $\sec\frac{5\pi}{4}$ = _____

85. $\csc\frac{\pi}{6}$ = _____

86. $\arcsin\frac{1}{2}$ = _____ arcsin(x) range: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

87. $\sin\frac{\pi}{2}$ = _____

88. $\arctan 1$ = _____ arctan(x) range: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

89. $\cot\frac{\pi}{2}$ = _____

For # 90- 95, solve each of the equations for $0 \le x \le 2\pi$. Isolate the variable and find <u>all</u> of the solutions within the given domain. [EXACT SOLUTIONS = NO DECIMALS! (may contain π)].

90. $\sin x = -\frac{1}{2}$ (hint: 2 answers)

91. $\sin 2x = -\frac{\sqrt{3}}{2}$ (hint: 4 answers)

92. $4\cos^2 x - 3 = 0$ (hint: 4 answers)

93. $2\cos x = \sqrt{3}$ (hint: 2 answers)

94. $\sin^2 x = \frac{1}{2}$ (hint: 4 answers)

95. $\cos 2x = \frac{1}{\sqrt{2}}$ (hint: 4 answers)

For #96-109, solve for x (without a calculator). Show ALL steps that lead to your answer. *Some answers may contain exponential and logarithmic expressions.*

96. $3^{2x} = 27$

97. $5^{3x} = \frac{1}{125}$

98. $5 - 3^x = -40$

99. $2^{3x-4} = 5$

100. $\log(3x + 1) + 1 = 3$

101. $2\log(x + 1) = 5$

102. $\log 2x + \log x = 11$

103. $\log(7x + 1) = \log(x - 2) + 1$

104. $\ln 3x = 6$

105. $\ln(x - 1)^2 = 3$

106. $2 \ln 2x^2 = 1$

107. $e^x = 18$

108. $e^{3x} + 5 = 6$

109. $e^{x+1} = 3$

110
104. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

a. Use your calculator to find the relative maximum and minimum y-value of $f(x)$.

min = _____ max = _____

b. State the domain of $f(x)$ in interval notation.

D: _____

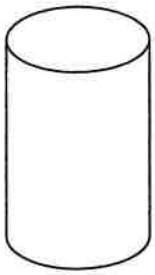
c. State when the function is increasing and decreasing. Write in interval notation.

increasing: _____ decreasing: _____

111.
105. A rectangular sheet of tin measures 20 inches by 12 inches. Suppose you cut a square out of each corner and fold up the sides to make an open-topped box. What size square should you cut out in order to maximize the volume of the box? Show all work to earn credit.

_____

112.
106. You have been asked to design a cylindrical can that will hold 1000 cubic centimeters. What dimensions (height and radius) will use the least amount of material?



$r =$ _____ $h =$ _____